Intermediation in Networks *

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Abstract

This paper studies decentralized trade in networked markets with intermediaries using a stochastic model of multilateral bargaining in which players compete on different routes through the network. The paper characterizes stationary equilibrium payoffs as the fixed point of a set of intuitive value function equations and studies efficiency and the impact of network structure on payoffs. In equilibrium players will never pass on an efficient trade opportunity, but they may trade in situations where delay would be efficient. With homogeneous surplus, the payoffs for players who are not essential to a trade opportunity go to zero if there is at least one essential player as trade frictions vanish.

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1 Introduction

This paper studies a network model of decentralized markets with intermediation. The network perspective, which puts the structure of connections between trading parties at the heart of the analysis, is particularly appropriate for the study of markets in which existing relationships matter for the interaction of economic agents. Many settings can be usefully thought of as networked markets, including markets explicitly relying on transport networks (pipelines, rail networks, ports) as well as markets where the connections are less tangible and consist of relationships such as in over-the-counter (OTC) trading in financial markets, international trade, and complex consumer goods, for example, in real-estate and insurance. In the latter markets, connections take the form of relationships built on trust, a history of previous interaction or having sufficient information about trading partners. For example, in financial markets, a pre-existing relationship helps OTC traders to manage their counterparty risk exposure, overcome reputational concerns and ensure that effective collateral provisions are in place.

In these relationship-based markets we often find intermediaries in the form of dealers, brokers and market makers that provide intermediation services for actors that do not trade directly with each other. The need for such intermediation arises naturally in network settings whenever there are opportunities for trade involving two parties that do not have a direct relationship, preventing them from dealing with each other directly. They may then nonetheless exploit their opportunities for mutual trade by engaging indirectly and involving one or more intermediaries, that provide the necessary chain of relationships to make the trade feasible.

In this paper I propose a modeling approach that explicitly incorporates a network perspective on intermediation activity. The approach brings into focus the role and the value of relationships used by third parties to facilitate transactions between economic agents who otherwise might lack the opportunity to conduct trade directly. Specifically, I present a dynamic model of multilateral bargaining and exchange in a network setting with intermediation. Each period, a random matching process selects a *route*, that is, a group of players connecting two trading parties via connecting intermediaries in the network. One — randomly selected — player on the route can make a proposal to the other players. If it is accepted, the trade is implemented and, importantly, the game ends, that is, there is no replacement. If at least one player on the route rejects, a new route and proposer are drawn. I show that the model has a stationary subgame perfect equilibrium in which payoffs are characterized through an intuitive set of value function equations and use this to study efficiency and the sharing of surplus between parties. The equilibrium payoffs illustrate the effect that competition between intermediation routes has on the payoffs players can expect. Efficiency considerations come into play when different routes may offer different levels of surplus, resulting for example from variation in buyer valuations or trade costs. The question is then to find the correct routes to trade on. I show that while in equilibrium players never unduly delay trade and refuse to agree on a route where trade would be efficient, there can exist instances where players agree to trade on inefficient routes when delay would be welfare improving. This inefficiency arises from the strategic advantage for players who can trade across multiple routes with alternative players. They can increase their own payoffs relative to those who are in competition with each other. Players thus have an incentive to keep in play multiple routes, even if not all of them are efficient for trade.

That markets for financial assets may be thought of as networks is powerfully exposed by looking at the data on trades in such markets. Early work in this direction includes Upper and Worms (2004) and Craig and von Peter (2014) analyzing the German interbank market. Their data reveal a network in a core-periphery structure with many peripheral banks that do not trade directly with others but only through the well-connected intermediaries forming the core of the network. The model in this paper can be usefully seen to capture a market with such a core-periphery configuration: The sellers in the model represent banks in the periphery trying to access potential other buyers that are also in the periphery. As no direct connections exist between banks in the periphery, intermediaries from the core of the network are required to facilitate the trade. The model then offers useful predictions concerning the trading patterns across the network as well as the incentives for the banks to position themselves in the network.

While I refer to buyers and sellers throughout the paper, the model may usefully be applied to study other value-adding interactions between multiple actors, such as liquidity provision between banks, R & D cooperation between firms, the formation of joint companies by multiple entrepreneurs, government coalition formation in political economy settings, etc.

The model is built around a one-off trade opportunity. Once trade concludes, the game is over and there is no replacement, which is a key distinguishing factor relative to the existing literature such as Nguyen (2015) and Manea (2018). This assumption of no replacement approximates trade in thin, highly customized products, such as complex financial securities commonly traded in OTC markets. It contrasts with markets of more generic assets such as commodities or standard financial contracts where there may be many buyers and sellers in the market at the same time. Such trades are easier to move to standardized trading platforms and central counter-parties. Even then, players might still prefer to trade via preferred dealers as shown, for example, in J. Allen and Wittwer (2023). In addition, some alternative applications for the model, such as cooperations in R & D where the output is a patent or other innovation, tend to exhibit very notable first-mover advantages, such that the first successful coalition captures most of the available gains. The assumption of a one-off opportunity without replacement describes well such first-mover advantages.

The paper is structured as follows. The next Section 2 provides the literature context for the research questions investigated. Section 3 sets out the model and Section 4 characterizes the equilibrium. An analysis and key results of the paper concerning efficiency and the relationship between structural features and payoffs are presented in Sections 5 and 6. Section 7 concludes.

2 Literature Context

This paper presents a contribution to the fast-growing literature on trade in networks and, in particular, the analysis of intermediation in such networks.

The provision of intermediation services and middlemen activities which this paper investigates in a network setting has been investigated in other non-structural frameworks by several authors, with overviews provided in Bose (2001) and Spulber (1999). Intermediaries have been credited with a number of different functions, including the provision of immediacy (Demsetz, 1968), or acting as a screening device between different types of traders that might be prevented from engaging directly with each other as in Bose and Pingle (1995) or Brusco and Jackson (1999). In the latter, an intermediary arises endogenously to overcome inefficiencies in trade across competitive markets. A seminal paper in this literature is Rubinstein and Wolinsky (1987). They investigate a setting with three types of players: buyers, sellers, and middlemen. Trade is conducted based on stochastic pairwise matching and a steady state equilibrium is derived.[1] A key insight of that paper is that the outcome of trade and the terms of trade depend on whether the middleman takes ownership of the good from sellers or work on a consignment basis. In the first case, the market is biased in favor of buyers, whereas in the second case symmetry between parties is restored. Duffie, Gârleanu, and Pedersen (2005) study a search and matching model for OTC markets. They analyze a model in which trading opportunities arise endogenously and study amongst others the implications of greater competition for intermediation services. As in Rubinstein and Wolinsky (1987), the model does not capture heterogeneity in the connections that traders may have to the intermediaries and amongst the intermediaries itself.

In contrast to the work cited above, structural features are at the core of a fast-growing literature on exchange in networks with numerous recent contributions. Seminal early works in this field include Corominas-Bosch (2004) on bargaining in networks and the exchange model in Kranton and Minehart (2001). Both adopt a bipartite networks approach, precluding an analysis of intermediation. More recent contributions in this direction include Manea (2011), Elliott (2015), Polanski (2007) and Polanski and Vega-Redondo (2017). Models which allow for multiple steps in trading come in two distinct flavors. Gale and Kariv (2007), Manea (2018) and Gofman (2017) all consider a trading protocol in which the good travels from seller to buyer in a stepwise fashion, with traders interacting bilaterally at each step. The paper by Nava (2015), which studies quantity competition instead of an explicit bargaining setting, arguably also falls into this category as intermediaries benefit from double marginalization, that is, extracting rents both upstream from sellers and downstream from buyers.

[1]In steady state equilibrium the outflow of pairs of traders who conclude a trade is exactly balanced by an exogenously given inflows of players.

A different group of papers including Blume et al. (2009), Polanski and Lazarova (2015) and Nguyen (2015) allow for simultaneous multilateral interaction, which is also the approach I adopt in this paper. The key distinction of the current work is that contrary to Blume et al. (2009) I consider an explicit bargaining protocol whereas they consider price-setting intermediaries (whom they call "traders"). Furthermore, different from Nguyen (2015) and Polanski and Lazarova (2015) I focus on a setting without replacement, that is, an environment where parties that conclude a trade are not replaced by replica players. My model is therefore more suitable to study markets where "trade opportunities" are just that: opportunities that ought to be taken, where a decision not to act carries an opportunity cost via the risk of missing out.[2] The model thus offers insights for real world markets where trade opportunities are not limitless, which arguably is the case in many relationship-based markets, including for financial and non-financial assets as well as interactions in which players collaborate to conduct a project together, for example, an R & D joint venture. The assumption of no replacement has significant implications for the equilibrium predictions. For example, competition between multiple intermediaries to facilitate the trade is significantly tougher than in models with replacement.

The literature on financial networks employs network tools to analyze various aspects of financial markets, including risk sharing and contagion amongst financial institutions. An overview is provided in F. Allen and Babus (2009). Recent contributions in Babus and Hu (2017) and Farboodi (2023) provide a network perspective to OTC trading and investigate the incentives for financial institutions seeking to exchange assets to form relationships for trading and intermediation.

[2] And even if a new, similar opportunity were to arise, the opportunity cost argument still applies as long as players are not prevented from taking part in more than a single trade.

Finally, at a technical level, this paper employs the framework of stochastic bargaining games with perfect information analyzed in detail in Merlo and Wilson (1995, 1998) and extends it for use in analyzing games on networks. One contribution of my paper to this literature is to identify a different source of equilibrium inefficiency in such stochastic bargaining settings, which does not arise in the setting of Merlo and Wilson (1995, 1998) as their model does not allow for the set of players bargaining to change between periods. This feature is crucial in the network setting I study as the changing coalitions correspond naturally to different trade routes and from a strategic perspective introduce the possibility that players may be excluded from the bargaining table, which has important implications for the analysis.

3 Model

This section presents a model in which players bargain over a surplus on a network. We consider a setting in which a network of relationships describes the possibilities for players to interact. Players have access to an opportunity that generates surplus, for example, generated by transferring an asset from a seller with a low valuation for the asset to a buyer with a high valuation, in the spirit of Duffie, Gârleanu, and Pedersen (2005). Players are matched along the network of existing relationships and bargain over the allocation of the available surplus within groups that form feasible trade routes. The bargaining protocol allows for the random selection of trade routes as well as the identity of the proposer, incorporating the notion of competition between different alternative trade routes.

The model I present here includes a number of stark simplifying assumptions, for example, concerning the underlying matching and bargaining protocol. These have been imposed to make the exposition as clean and transparent as possible. The general insights remain valid under less restrictive assumptions on many elements of the model.

Players Players are denoted by the set $N = \{1, 2, \ldots, n\}$. There is a set of sellers labelled $A = \{A_1, A_2, \ldots\} \in N$ who have access to a single, indivisible good that they can sell to any one of a set of buyers $B = \{B_1, B_2, \ldots\} \subset N$ such that $B \cap A = \emptyset$.[3]

Trade Network Players interact according to an undirected network denoted by $g = (N, E)$ where the set of edges $E \subset \left\{\left(i, j\right) : i \neq j \in N\right\}$ describes the set of feasible bilateral interactions. We restrict attention to networks where buyers only connect to intermediaries or sellers, and sellers only connect to intermediaries and buyers. Intermediaries can connect to buyer, sellers, and other intermediaries. This assumption is consistent with the core-periphery structure that has been found in many network markets such as financial networks (Li and Schürhoff, 2018; Craig and von Peter, 2014). A seller and a buyer can trade with each other other if and only if there exists a path in g between them. As will be described in greater detail below, trade between two nodes that are only indirectly connected is feasible through intermediaries if there exists at least one path between them. I assume that the network is connected.[4]

Routes A route $R \subseteq N$ between a pair of nodes i and j is an unordered set of nodes such that there exists a path $(i_1, \ldots, i_K)$ with $(i_k, i_{k+1}) \in E \ \forall \ k = 1, 2, \ldots, K$, $i_1 = i$, $i_K = j$ and each node in the sequence distinct. As the network is

[3]The labels of buyers and sellers can be reversed without consequence for further analysis. The key simplification of the model is that there is just one trade opportunity, and one node is involved in all possible coalitions that can realize the opportunity.

[4]This assumption is without loss of generality here as disconnected players simply cannot trade.

connected by assumption there exists at least one path in the network g between each buyer/seller pair. We call such a set of nodes connecting a seller A_i and a given buyer B_j a *route*. Each route R_l has a surplus v_l attached to it reflecting the surplus available, that is, buyer valuation less any costs including the seller's reservation price. Depending on the network g, for any given buyer-seller pair there may be multiple feasible routes involving different combinations of intermediaries.[5]

Matching and Bargaining Protocol The model operates in discrete time. In each period players are matched and bargain under a stochastic route selection and bargaining protocol building on Merlo and Wilson (1995) as follows.

Each period one trade route including a seller and a buyer is activated and an order of play for players on this route is randomly determined. Based on this draw, players who are on the route bargain according to the order prescribed within the state, with the first acting as proposer.

Formally, in each period a state s from finite state space S is selected by a Markov process $\sigma = (\sigma_0, \sigma_1, \sigma_2, \ldots)$. A state s contains information about three elements of the model:

i. The active buyer $B(s) \in B$.

ii. The route $R(s) \subseteq N$ connecting the pair of players who have the trade opportunity with associated valuation $v(s)$, representing the surplus available in state s if there is agreement.

iii. A permutation $\rho(s)$ on $R(s)$ which denotes the order in which players move through the bargaining protocol. $\rho_i(s) \in N$ denotes the player

[5] One may restrict attention to shortest paths or geodesics only, but this restriction is not essential for the analysis.

moving in i-th position. Following Merlo and Wilson (1995) we denote by $\kappa(s) \equiv \rho_1(s)$ the first mover in the order.

We take the set of states S to span all feasible trade routes in g as well as for each route all permutations of players on that route. Furthermore, to simplify the exposition I assume that σ is time homogeneous, such that $\sigma_t = \sigma_{t'} \; \forall \; t, t'$ and each period's draw is independent of the previous period's state. The independent ex-ante probability of state s is denoted $\pi(s)$. Finally, we assume each $s \in S$ is drawn with strictly positive probability. Thus, every route is selected, and every player is called upon as proposer with positive probability.[6]

On realization of state s, player $\kappa(s)$ may propose an allocation or pass. If a proposal is made, this takes the form of a vector $x \in \mathbb{R}^n$ such that $x_i \geq 0 \; \forall \; i \in R(s)$. We assume that any players not on the route are offered zero payoffs, that is, $x_i = 0 \; \forall \; i \notin R(s)$.[7] For the proposal to be feasible requires that $\sum_{i \in R(s)} x_i \leq v(s)$. A proposal x thus represents a split of available surplus amongst the players on the route, allocating a nonnegative share x_i of the surplus to each player in $R(s)$ and a zero share of the surplus to all excluded players. The other players on the route then respond sequentially in order given by $\rho(s)$ by accepting or rejecting the proposal. This process continues until either (i) one player rejects proposal x or (ii) all players in $R(s)$ have accepted it.

If all responders accept x, the proposed allocation is implemented and the

[6] The assumption of independence allows me to dispense with conditioning on the current state whenever expectations about future realizations are formed and follows standard random proposer bargaining games. However, a more general Markov process, for example one that preferentially selects among states on the route that was most recently active, would leave results qualitatively unaffected as long as it is ergodic.

[7] This assumption is natural given the restriction that only players on the active route get to respond to a proposal. Furthermore any accepted proposal that offered a strictly positive payoff to excluded players would not be optimal in a stationary equilibrium.

game ends. If the proposer passes or at least one responder rejects the proposed split, the bargaining round ends, and the game moves to the next period in which a new state s' consisting of both a route $R(s')$ and a new order of play $\rho(s')$ is drawn and the bargaining process is repeated. This sequence is continued until an allocation is accepted by all players.

Information Structure All players observe the realized states and all actions taken by other players.

Payoffs Payoffs are linear in the share of surplus allocated, with common discount factor $\delta \in (0,1)$. If proposal x is accepted in period t, player i receives utility:

$$u_i(x) = \delta^t x_i \tag{1}$$

We assume that the surplus to be allocated is bounded above such that $u_i(x) \to 0$ as as agreement time $t \to \infty$.

The model forms an infinite horizon dynamic game of complete information. Players take a decision in two distinct roles: as proposer and as responder. As proposer, a player either passes or suggests a split of surplus on a given route conditional on the route selected and being selected as proposer. As responder, players have to decide whether to accept or reject a proposed surplus division. A responder's decision is conditioned on the selected route and proposer as well as the surplus division on the table. A history is defined by a sequence of realized states and actions taken by players. A strategy specifies a feasible action at every possible history when a player must act.

It is worth discussing briefly the assumptions on matching and bargaining underlying the model. The matching protocol with cost of delay is intended to capture the frictions inherent in the trading protocol. For example, search frictions may arise from identifying a possible counterparty or from eliciting their willingness to trade. Furthermore, there may be frictions arising from intermediaries, such as dealers in financial markets, identifying the regulatory implications of participation as described in Duffie (2010). Importantly, the result of the frictions is that there exists an opportunity cost of not concluding a deal in any given state. In the model, this opportunity cost consists of the utility cost of delay, the cost of ending up in a weaker bargaining position from not being a proposer in a future state, and — crucially for this paper — the cost of potentially being left out of a future trade entirely. In addition, the simple frictions in the model may be thought of as a summary of more complex trading protocols, such as the "request for quotes" protocol as studied in Burdett and Judd (1983), which is frequently employed in real-world OTC markets. Burdett and Judd (1983) show that in their model the more complex trading protocol can yield payoff outcomes identical to a much simpler bargaining model.

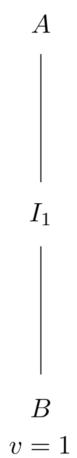
Bargaining in the model is multilateral and adopts the unanimity rule: the good remains with the seller unless agreement with all intermediaries on the selected route to the buyer has been reached. Thus, the model is applicable to markets in which intermediaries act as a broker or agent, facilitating trade between buyers and sellers, rather than ones in which intermediaries take possession of the good, acting as a dealer instead.[8] Considerations which arise in markets described by a

[8]Reporting of corporate bond markets suggests that in the wake of the 2008 financial crisis brokers increasingly showed the behavior implied in the model: "In the wake of the financial crisis and ahead of tighter regulatory constraints, large Wall Street dealers have become far less willing to hold the risk of owning corporate bonds, known in market parlance as 'inventory,' to facilitate trading for their clients. Instead, they are increasingly trying to match buyers and sellers, acting more as a pure intermediary, rather than stockpiling bonds and encouraging a liquid market for secondary trading." Source: "*Slimmer bond inventories as dealers reduce risk*", Financial Times, November 8, 2011.

good "traveling" along the route, with intermediaries assuming ownership, such as questions of hold-up (intermediaries being in possession of the good but not intrinsically valuing it) or counterparty risk associated with disappearing resale opportunities, thus, remain outside the model.[9]

3.1 Example State Space

To illustrate the model and the workings of the matching and bargaining protocol, consider the network displayed in Figure 1. There is just one feasible trade routes generating a surplus of 1. The trade route consists of one seller A, one intermediary I and one buyer B. There are six feasible permutations of the three players on the route. In total, there are thus six states as enumerated in the adjacent table.



s	$\pi(s)$	$R(s)$	$\rho_1(s)$	$\rho_2(s)$	$\rho_3(s)$	$\kappa(s)$
1	1/6	$\{A, I, B\}$	A	I	B	A
2	1/6	$\{A, I, B\}$	A	B	I	A
3	1/6	$\{A, I, B\}$	I	A	B	I
4	1/6	$\{A, I, B\}$	I	B	A	I
5	1/6	$\{A, I, B\}$	B	A	I	B
6	1/6	$\{A, I, B\}$	B	I	A	B

Figure 1: Example Network and State Space with a Single Trade Route

[9]See the discussion in Rubinstein and Wolinsky (1987) concerning the difference between middlemen taking ownership of the good and acting on consignment. Models exploring trade in networks in which the good travels on a bilateral basis from seller to buyer are analyzed in Gofman (2017), Condorelli, Galeotti, and Renou (2017) and Li and Schürhoff (2018).

4 Equilibrium Payoffs

This section develops the equilibrium analysis of the model. We restrict attention to stationary subgame perfect equilibria (SSPE), that is, subgame perfect equilibria consisting of strategies which condition on payoff relevant histories only: the state (selected route and order of proposals), and the offer on the table in the given period. Stationary equilibrium payoffs are characterized as a fixed point to an intuitive system of recursive equations. All proofs in this as well as subsequent sections are collected in the appendix.

Let f be an expected payoff vector defined by $f = E_\sigma\left[f(s)\right] \in \mathbb{R}^n$, where $E_\sigma\left[\cdot\right]$ is the expectation operator for the probability distribution over the state space and $f(s) \in \mathbb{R}^n$ denotes the vector of payoffs for players in state s. Define an operator $\mathbb{A}$ on payoff f which maps from $\mathbb{R}_+^{n\cdot|S|}$ to $\mathbb{R}_+^{n\cdot|S|}$ such that:

a. (**Agreement**) If $v(s) > \delta \sum_{j\in R(s)} E_\sigma\left[f_j(s')\right]$:

$$\mathbb{A}_i(f)(s) = \begin{cases} v(s) - \delta E_\sigma\left[\sum_{j\in R(s)\setminus i} f_j(s')\right] & \text{for Proposer } i = \kappa(s) \\ \delta E_\sigma\left[f_i(s')\right] & \text{for Responder } i \in R(s)\setminus\kappa(s) \\ 0 & \text{for Excluded } i \notin R(s) \end{cases} \tag{2}$$

b. (**Delay**) If $v(s) < \delta \sum_{j\in R(s)} E_\sigma\left[f_j(s')\right]$:

$$\mathbb{A}_i(f)(s) = \delta E_\sigma\left[f_i(s')\right] \;\; \forall\, i \in N \tag{3}$$

c. (**Mixing**) If $v(s) = \delta \sum_{j \in R(s)} E_\sigma \left[f_j(s') \right]$:

$$\mathbb{A}_i(f)(s) = \begin{cases} \delta E_\sigma \left[f_i(s') \right] & \forall\, i \in R(s) \\ \left\{ x : x \in \left[0, \delta E_\sigma \left[f_i(s') \right] \right] \right\} & \forall\, i \notin R(s) \end{cases} \tag{4}$$

The payoff operator $\mathbb{A}(f)$ distinguishes three cases depending on $v(s)$, the surplus in state s. These can be interpreted as follows:

a. (**Agreement**) If the available surplus $v(s)$ exceeds the total expected value of moving to the next period for players on the selected route ($\delta E_\sigma \left[\sum_{j \in R(s) \setminus i} f_j(s') \right]$), then $\mathbb{A}(f)$ assigns to the proposer a payoff that extracts from responding parties on the selected route all surplus over and above their discounted expected value of moving to the next period given by $\delta E_\sigma \left[f(s') \right]$, leaving zero to players not included on the route.

b. (**Delay**) If the available surplus $v(s)$ is less than the expected value of moving to the next period for players on the selected route, then $\mathbb{A}(f)$ assigns that payoff to each player.

c. (**Mixing**) If the available surplus $v(s)$ is equal to the discounted expected value of moving to the next period summed for players on the selected route, $\mathbb{A}(f)$ for these players is equal their discounted next period expected payoff. For excluded players the payoff is assigned to the interval in the real line between zero and their discounted expected next period payoff.

A stationary equilibrium payoff of the bargaining game is a fixed point of this correspondence. The proof follows standard approaches and is presented in the appendix.

Proposition 1. *f is an SSPE payoff if and only if $\mathbb{A}(f) = f$.*

Existence of an equilibrium payoff vector can be established using Kakutani's fixed point theorem on the operator $\mathbb{A}$.

Proposition 2. *There exists an SSPE payoff f.*

The equilibrium payoff is supported by a strategy profile in which every player adopts a strategy with the following standard properties. When responding a player accepts any offer which gives her at least the discounted expected next period payoff and reject otherwise. If proposing, she offers every responder their outside option if the residual amount is strictly larger than the proposer's discounted expected next period payoff. If the residual is strictly less, the proposer passes with probability one. In case of indifference the proposer makes an offer as above with probability between zero and one. We discuss the role of such "mixing" states further below.

Proposition 1 allows the analysis of equilibrium outcomes and payoffs for all possible trade networks and buyer valuations based on a set of equations describing value functions in a recursive manner. We will exploit the characterization to study efficiency and the impact of network structure on equilibrium outcomes in subsequent sections. At this point it is worthwhile to emphasize the implications of the "no replacement" assumption on equilibrium payoffs. Proposition 1 implies that excluded players receive a zero payoff in states of agreement while they can have a positive expected payoff in states of disagreement. The potentially positive payoff in disagreement states reflects the fact that the players excluded in the current state may be included in successful negotiations in a future period.

The zero payoff for excluded players in case of agreement presents a significant difference to models with replacement (for example, Nguyen (2015) and Polanski and Lazarova (2015)) in which players who do not take part in a trade that is concluded simply wait for the next period to be offered an essentially unchanged environment opportunity. It significantly intensifies the competition between different trading routes as they vie to be included in the group that reaches agreement. Section 6 provides further analysis on this topic.

In addition, the fact that excluded players in agreement states earn zero payoffs implies that an application of the proof of payoff uniqueness in every state in Merlo and Wilson (1995) based on the contraction principle fails. In their proof, Merlo and Wilson (1995) exploit the fact that the payoff for each player i in state s is bounded below by $\delta E_\sigma \left[f_i(s') \right]$, which allows them to show that the equilibrium operator forms a contraction when using a suitable norm. In the model in this paper, payoffs for an excluded player jump to zero under the operator $\mathbb{A}$ if player i is excluded and there is agreement in state s. This implies that in some settings the Merlo and Wilson (1995) norm for the payoff vector of all players can decrease below the sum of their discounted expected future payoffs, which breaks the argument. A similar approach to show uniqueness is applied in Polanski and Lazarova (2015), which likewise does not apply to the model with excluded players. Indeed, the fact that these proofs of equilibrium payoff uniqueness for each state do not work is unsurprising, given that in equilibria in mixed strategies with two or more states on a route, different combinations of agreement probabilities across these mixing state may support the same vector of expected equilibrium payoffs.

4.1 Example Equilibrium Payoffs

To illustrate the equilibrium payoff characterization of Proposition 1, we return to the example in Figure 1. Given that the available surplus in every state is one, we conjecture that agreement will take place in every state, compute the resulting payoffs, and verify the agreement decision later. Under the conjecture, buyer A will receive a "responder" payoff of $\delta E_\sigma\left[f_A(s')\right]$ in four out of six states $(3-6)$. Thus, $f_A(s)=\delta E_\sigma\left[f_A(s')\right]$ for $s\in\{3,4,5,6\}$. When proposing, A will receive the residual surplus after offering just enough to I and B to make them accept. Thus, $f_A(1)=f_A(2)=1-\delta E_\sigma\left[f_I(s')\right]-\delta E_\sigma\left[f_B(s')\right]$. Plugging these expressions into the expansion of $E_\sigma\left[f_A(s')\right]$ yields:

$$E_\sigma\left[f_A(s')\right]=\frac{4}{6}\delta E_\sigma\left[f_A(s')\right]+\frac{2}{6}\left\{1-\delta E_\sigma\left[f_I(s')\right]-\delta E_\sigma\left[f_B(s')\right]\right\} \tag{5}$$

By symmetry, identical expressions characterize $E_\sigma\left[f_I(s')\right]$ and $E_\sigma\left[f_B(s')\right]$ and in equilibrium all three players receive the same payoff. We can solve Equation 5 for $E_\sigma\left[f_A(s')\right]=\frac{1}{3}$. Finally, the solution is consistent with our conjecture about agreement behavior: because we have $\sum_{i\in R(s)}\delta E_\sigma\left[f_i(s')\right]=\delta<1\ \forall\ s\in S$ agreement in all states is indeed optimal for every player.

5 Efficiency

This section discusses the efficiency properties of the equilibrium of the bargaining game. Efficiency is achieved by adopting an optimal stopping rule which implements agreement in states offering sufficiently high surplus and delays otherwise.

Let $\phi(s):S\rightarrow[0,1]$ describe a function that for each state $s\in S$ denotes the probability of "stopping". Stopping implies that the surplus $v(s)$ is collected and the game ends. Not stopping implies that one period passes and a new state is drawn. Given independence of the realizations of s across time, the total surplus

$w(\phi)$ associated with a stopping rule ϕ is computed recursively by the expression

$$w(\phi) = \sum_{s \in S} \pi(s) \left\{ \phi(s)v(s) + \left[1 - \phi(s)\right] \delta w(\phi) \right\} \tag{6}$$

The optimal stopping rule ϕ^* is defined as:

$$\phi^* = \arg\max_{\phi} w(\phi) \tag{7}$$

Denote w^* the ex-ante expected total surplus that can be derived under the optimal stopping rule ϕ^*. By the principle of optimality, the efficient stopping rule ϕ^* satisfies a threshold rule for all $s \in S$ that collects the available surplus $v(s)$ if it is larger than w^* and passes otherwise:

$$\phi^*(s) = \begin{cases} 1 & \text{if } v(s) > \delta w^* \\ \phi : \phi \in [0,1] & \text{if } v(s) = \delta w^* \\ 0 & \text{if } v(s) < \delta w^* \end{cases} \tag{8}$$

The efficiency benchmark suggests two possible sources of inefficiency: there may be too much trade or too little. Too much trade is conducted if the parties involved in bargaining on a route agree to an allocation in a state in which it would be efficient to delay. There is too little trade if the parties do not agree on an allocation in a state where trade would be strictly efficient in the sense that available surplus strictly exceeds what could be gained from waiting. I will show that the SSPE of the game specified does not exhibit the latter type of inefficiency but is subject to the former.

Proposition 3. *In any SSPE players reach agreement with probability one in all states in* *which agreement is strictly efficient.*

Proposition 3 implies a corollary for the simplified setting in which all feasible routes generate the same surplus v. In this case, $w^* = v$ and consequently efficiency demands that trade be concluded immediately without delay.

Corollary 4. *If $v(s) = v\ \forall\ s \in S$, in any SSPE trade is conducted immediately and the equilibrium outcome is efficient.*

A necessary condition for delay in this model is thereby the heterogeneity of surplus across different routes.

Proposition 3 also implies that trade is concluded even along intermediation routes which may involve relatively large numbers of intermediaries when shorter, more direct routes are available. Thus, an intuitive prediction that it might be better for buyer and seller to delay trade in such situations to avoid splitting the surplus with too many additional parties does not hold. This is because payoffs for intermediaries on the longer route are endogenously adjusted downwards in equilibrium, reflecting the constraint exerted by the presence of the shorter route. Thus, in this model there is no "strategic" cost from additional intermediaries per se. What matters for whether a route is actively traded over is the surplus it generates. This feature is an important implication of the model consistent with experimental results reported in Choi, Galeotti, and Goyal (2017).

Can trade occur too early in equilibrium? Yes, as I will illustrate in a variation of the example seen above. Consider a setting with a single seller and two possible routes, each with one intermediary and one buyer, illustrated in Figure 2. The low valuation route generates a surplus of 1 while the high valuation route generates a surplus of $v \geq 1$. Assume as above a uniform stochastic process such that every route is selected with probability $1/2$ and along each route every player is selected with equal probability. Thus, each route is played half of the time and conditional on a route being selected each of the three players is proposing with equal probability.

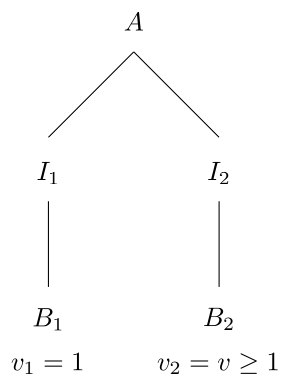


Figure 2: Network with Two Asymmetric Intermediation Routes

The efficient outcome in this case involves either trade along both routes or trade along the high value route with valuation v only, depending on the discount factor δ. Specifically, comparing expected total payoffs we can derive a first threshold discount factor, $\delta^* = 2/(1+v)$, at which delay and agreement on the low value route generate the same payoff. For $\delta > \delta^*$ efficiency requires trade to take place only along the high value route. In contrast, the vector of equilibrium payoffs is such that agreement takes place in low value states with positive probability for a range of $\delta > \delta^*$. To see why consider payoffs in a hypothetical equilibrium in which trade takes place with the low valuation buyer with probability zero. In this case, $E_\sigma\left[f_{B_1}(s)\right] = E_\sigma\left[f_{I_1}(s)\right] = 0$ as this route would never be actively traded on. For the players on the high value route (seller as well as the buyer and intermediary) the payoff equations would then be symmetric like the example in Figure 1 and can be solved for $E_\sigma\left[f_A(s)\right] = E_\sigma\left[f_{B_1}(s)\right] = E_\sigma\left[f_{I_1}(s)\right] = v/(6-3\delta)$.

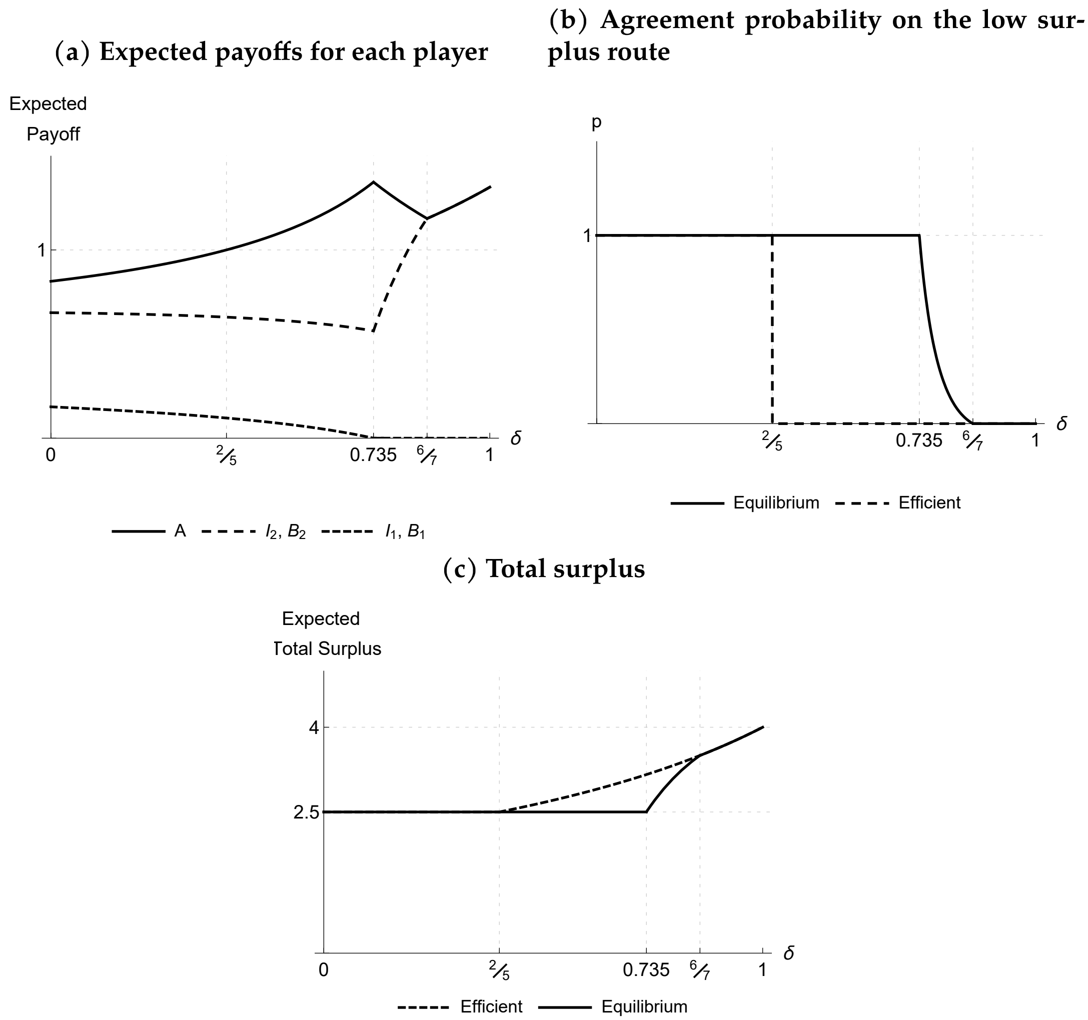
However, for δ smaller than a second threshold discount factor, $\tilde{\delta} = 6/(3+v)$, this solution would imply $\delta E_\sigma \left[f_S(s) \right] < 1$, meaning that the seller's discounted expected next period payoff would be less than the surplus available on the low valuation route. The seller would then have a profitable deviation to offer some $\epsilon > 0$ to the buyer and intermediary on the low valuation route and the responders would accept the offer. Thus, for $\delta < \tilde{\delta}$ in every stationary equilibrium trade occurs on the low surplus route with strictly positive probability.

Note that $\delta^* < \tilde{\delta}$ and thus there is an interval of discount factors with strictly positive measure in which equilibrium payoffs will be such that they imply trade with positive probability with the low valuation buyer — despite this outcome being inefficient. Indeed, as δ increases within the interval $\left(\delta^*, \tilde{\delta}\right)$ we observe that starting from $\delta > 2/5 \cdot \left(5 - \sqrt{10}\right) \approx 0.735$ the equilibrium involves mixed strategies such that trade occurs on the low value route with a probability that is positive, but strictly less than one. The intuition behind this property of the equilibrium is that the seller is effectively keeping the low-value route "in play" to maintain her strategic advantage relative to the high-value route.

Figure 3 summarizes the workings of the example with $v = 4$ by plotting the probability of agreement in low valuation states (Figure 3b), the total surplus (Figure 3c), and equilibrium expected payoffs for each player (Figure 3a) against the discount factor δ. The threshold discount factor δ^* above which trade on the low value route becomes inefficient is $2/5$ in this case. As Figure 3b illustrates, in equilibrium trade occurs with probability one for an interval above this and then declines smoothly, hitting zero at $6/7$. In between these two values, the expected total surplus realized in equilibrium is below the efficient one (Figure 3c).

Two further points are worth noting about the expected payoffs of the seller and the downstream players (buyer and intermediary) as shown in Figure 3a. First, for $\delta > 6/7$ we see the payoffs for the seller and the downstream players on the high

Figure 3: Equilibrium with inefficient agreement



value route overlapping, reflecting the "strategic symmetry" of the three players whenever only the high value route is traded on. Each of the three players propose with equal probability and therefore in equilibrium their expected payoffs will be equal as well. Second, for $2/5 < \delta < 6/7$ the chart shows higher payoffs for the seller, which illustrates the "strategic asymmetry" that results from the seller making active use of her outside option of trading on the low value route.

The "too much trade" inefficiency identified here can be usefully thought of as a hold-up problem: from an efficiency perspective the seller should "invest" by delaying in the low surplus state, accessing the social surplus of higher expected valuation. However, if the seller adopted such a strategy, the symmetry between the seller and the high valuation buyer and intermediary would result in equal expected payoffs for all three players which leaves the seller worse off. Efficiency in this case could be restored were the intermediary and buyer on the high valuation route able to commit to compensating the seller for the delay decision by promising a higher share of the surplus in the high value states. However, an SSPE does not permit strategies that could implement such a promise.

Looked at from another perspective, the inefficiency can also be regarded as the result of the seller's privileged position when there are two alternative routes available and her unwillingness to give up the payoff benefits that result from having such alternative sources of supply, even if it offers only an inefficiently low surplus. Consider a modification of the example where we simply remove the low valuation route, that is remove players I_1 and B_1, and replace the associated states in the state space with automatic "delay", that is, the ex ante probability of drawing the high value route remains as before. In that case with only the high value route available, the trading outcome would be efficient, even though we have removed opportunities to trade. In other words, a setting with an additional trade route, a "thickening" of the market, can lead to a less efficient outcome than the same setting without that route.[10]

Merlo and Wilson (1995) present a two-player example in their model without excluded players in which there is equilibrium agreement in the first period even though the available surplus is small in that period and delay would be Pareto im-

[10]While overall welfare is lower with the additional route, the seller payoff is higher, which suggests that traders in these markets may have an incentive to (over-) invest in creating competing routes, although a formal model of network formation would be required to analyze these incentives appropriately (see also Elliott, 2015).

proving (see Figure 2 in Merlo and Wilson (1995)). The reason for the early agreement in their setting lies in the construction of the state space with two possible states in the second period, both of which are absorbing, that is, whoever proposes in that state will also be the proposer for all periods afterwards. In order to avoid the risk of losing the right to propose for all future periods, both players are willing to agree in the first period even though delay would permit Pareto improving outcomes. The example presented in this paper differs from the one in Merlo and Wilson (1995) in at least two ways. First, it illustrates that inefficiently early agreement can also occur a more regular state space without relying on absorbing states, as soon as one allows for players being excluded in certain states, which is the key feature in the my paper. Second, while in the example in Merlo and Wilson (1995) inefficient agreement occurs with probability one and to avoid outright entering a disadvantageous future state, the network setting with excluded players in my paper allows for mixed equilibria with agreement probabilities between zero and one in the low surplus states. Players employ such mixed strategies not to prevent arriving at the future state, but instead to influence the implied bargaining strength in those future high surplus states, trading off the loss in the overall size of the surplus with the greater share they can secure.

6 Network Structure and Equilibrium Payoffs

This section considers the relationship between structural features of the trade network and equilibrium payoffs. One implication of Proposition 1 is that players excluded in a state where agreement is struck receive a zero payoff. Consequently, players who find themselves in such situations may be expected to have their bargaining power reduced. I investigate this question first by considering the way in

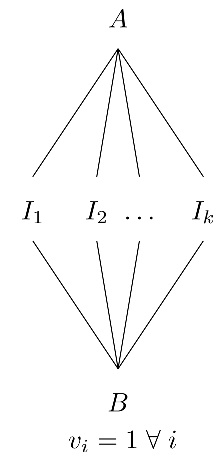


Figure 4: A setting with k intermediaries

which payoffs change as the number of competing intermediaries increases before deriving a more general result by considering the impact of being "essential" to a trade on payoffs. I restrict attention in the following to a setting in which all routes generate the same surplus in all states such that $v(s) = 1 \; \forall \; s$ to focus attention on the strategic competition between otherwise comparable routes.

6.1 Additional Intermediation Routes

To investigate the impact the number of intermediaries has on payoffs, consider first a simple setting with a single buyer and a set of k intermediaries that directly link to both a single seller and a single buyer for the asset (see Figure 4), each generating a surplus of 1. Expected equilibrium payoffs for the end-nodes A and B and any intermediary I_i are then given by $E[f_A]$, $E[f_B]$ and $E[f_{I_i}]$, respectively:

$$E[f_A] = E[f_B] = \frac{k - \delta}{k\,(3 - \delta) - 2\delta} \tag{9}$$

$$E[f_{I_i}] = \frac{1 - \delta}{k\,(3 - \delta) - 2\delta} \tag{10}$$

As expected, payoffs for end-nodes increase with the entry of additional intermediaries. Also, as $\delta \to 1$, payoffs for intermediaries go to zero. The ratio of the payoffs is given by $f_B/f_I = 1 + (k-1)/(1-\delta)$. At $k = 1$, the relative shares are equal and as k increases the ratio increases linearly at rate $1/(1-\delta)$.

6.2 Limit Payoffs on a Network with Competing Routes

The analysis in the previous section illustrates the impact of competition in a simple setting with a set of competing intermediaries each connected directly to both buyer and seller. One result of that analysis is that as trade frictions vanish in the limit intermediaries receive an expected payoff of zero. This section shows how the intuition derived from the simple example carries through to more general network structures.

Definition 1. *A player i is essential to a trade opportunity if $i \in R(s)$ $\forall$ $s \in S$.*

The definition applies the spirit of the approach adopted in Goyal and Vega-Redondo (2007) to the present model. Structurally speaking, a player is essential if he is located on all possible trade routes between the buyer and the seller of the good. Non-essential players can be circumvented and as a consequence they are competing for the business of intermediating the trade opportunity.

We will show that the property of being essential has implications for payoffs that players receive in equilibrium. In particular, in the limit as bargaining frictions decline, only essential players receive a payoff that is different from zero.

Proposition 5. *In an SSPE of the game with equal surplus in all states, if there exists at least one essential player, the payoff of any non-essential player i converges to zero as bargaining frictions diminish as $\delta \to 1$.*

Proposition 5 provides microfoundations for an analysis of competing intermediaries on networks and maps the intuitive Bertrand outcome into the bargaining setting investigated here. As such it provides a justification for the payoff structure used in Goyal and Vega-Redondo (2007), who investigate incentives for network formation in a setting with intermediation rents. While they assume that non-essential players receive zero payoff, justifying it as the kernel and core in a cooperative bargaining setup, the present analysis may provide some grounding for this assumption in a non-cooperative bargaining setting.

In addition, the result refines a prediction found in Choi, Galeotti, and Goyal (2017) and provides a theoretical underpinning for the experimental findings in that paper. In Choi, Galeotti, and Goyal (2017) the authors present a model of trade in a network with posted prices. The model exhibits multiplicity of Nash equilibria in settings with competing intermediaries, with strictly positive payoffs for these intermediaries in some equilibria and zero payoffs in others. The authors draw on this indeterminacy regarding the division of surplus in part to motivate an experimental investigation and they find that in a laboratory setting intermediaries that cannot be avoided extract a higher share of surplus. By contrast to the theoretical model in Choi, Galeotti, and Goyal (2017), Proposition 5 implies that non-essential intermediaries earn zero payoff in *every* equilibrium, consistent with their experimental evidence.

7 Conclusion

In this paper, I study a model of bargaining and exchange with intermediation on networks, extending the Merlo and Wilson (1995) framework as a tool to analyze stochastic bargaining games into a network setting. I characterize payoffs with a simple set of value function equations allowing the analysis of efficiency and the impact of structure on payoffs in equilibrium outcomes. I find that trade in settings with homogeneous valuations across all routes, trade is efficient. However, with heterogeneity of surplus across routes, there can be too much trade in the shape of inefficiently early agreement in equilibrium, arising from a potential hold-up problem. Competition between alternative routes is shown to reduce payoffs. For the case of equal surplus across routes and states, in the limit as bargaining frictions disappear, all players who are not essential to a trade opportunity receive zero payoffs in equilibrium. I have imposed several simplifying assumptions to offer a clean and transparent exposition of the main channels at work in my model. The same qualitative insights would likely remain valid if the model were generalized in a number of possible directions, including a more general stochastic process of selecting routes and proposers.

The present analysis suggests there is scope for future research in several directions. These include, notably, a more explicit study of the implications of the bargaining model for network formation identifying the incentives for players to invest in connections. The resulting predictions can then be compared to those in models with different payoffs structures including for example Babus and Hu (2017) and Goyal and Vega-Redondo (2007).

8 Appendix

8.1 Proof of Proposition 1

This section presents the proof of Proposition 1. The approach taken employs a standard argument adapted from Merlo and Wilson (1998). The proof of the proposition requires demonstrating that f is an SSPE payoff *if and only if* $\mathbb{A}(f) = f$.

Proof. Direction $\Rightarrow$*: "f is an SSPE payoff" implies "$\mathbb{A}(f) = f$"*

Consider an SSPE payoff f and fix a state s with $i = \kappa(s)$. Given f, it is a best reply for responder j to a given proposal x to reject if $x_j < \delta E_\sigma \left[f_j(s')\right]$ and to accept if $x_j > \delta E_\sigma \left[f_j(s')\right]$. This implies that i can earn $v(s) - \delta E_\sigma \left[\sum_{j \in R(s), j \neq i} f_j(s')\right]$ from making a proposal that is accepted and $\delta E_\sigma \left[f_i(s')\right]$ from passing. Thus, if $v(s) < \delta E_\sigma \left[\sum_{j \in R(s)} f_j(s')\right]$, the proposer will pass in a SSPE and $f_i(s) = \delta E_\sigma \left[f(s')\right] \ \forall \ i$. If $v(s) > \delta E_\sigma \left[\sum_{j \in R(s)} f_j(s')\right]$, i will make a proposal in an SSPE that is accepted, earning:

$$v(s) - \delta E_\sigma \left[\sum_{j \in R(s), j \neq i} f_j(s')\right] \text{ for } i \tag{11}$$

$$\delta E_\sigma \left[f_j(s')\right] \text{ for } j \in R(s) \setminus i \tag{12}$$

$$0 \text{ for } k \notin R(s) \tag{13}$$

If $v(s) = \delta E_\sigma \left[\sum_{j \in R(s)} f_j(s')\right]$, the proposer is indifferent with $f(s) = \delta E_\sigma \left[f(s')\right]$ again. This implies that in an SSPE an agreement can be reached with any probability between zero and one, which implies payoffs for any excluded player k that are in $\left[0, \delta E_\sigma \left[f_k(s')\right]\right]$. Thus $\mathbb{A}(f) = f$.

Direction $\Leftarrow$*: "$\mathbb{A}(f) = f$" implies "f is an SSPE payoff"*

Assume $\mathbb{A}(f) = f$. We show that f is an SSPE payoff by defining a suitable strategy profile and demonstrating that no player can be better off by unilaterally deviating. The strategy profile instructs proposers to pass unless $v(s) < \delta E_\sigma \left[\sum_{j\in R(s)} f_j(s')\right]$ in which case the proposer offers each responder j the $\left[f_j(s')\right]$. Responders will then accept, which yields $\delta E_\sigma \left[f_i(s')\right]$. Now, given payoffs f there is no incentive for any $j \in R(s) \setminus i$ to deviate and reject. For player i, there is no incentive to deviate as $f_i(s) \geq \delta E_\sigma \left[f_i(s')\right]$. Finally, for $k \notin R(s)$, the rules are such that no action is taken and thus there is no possibility for deviation to consider. Similarly, if $v(s) > \delta E_\sigma \left[\sum_{j\in R(s)} f_j(s')\right]$ given decision rules by responders, proposer i cannot benefit from deviating to a proposal that is accepted with positive probability. Finally, if $v(s) = \delta E_\sigma \left[\sum_{j\in R(s)} f_j(s')\right]$ the strategy profile instructs the proposer to make an acceptable proposal with positive probability $\phi(s)$ such that for excluded players k $\phi(s) \cdot E_\sigma \left[f_k(s')\right] = f_k(s)$ as required.

□

8.2 Proof of Proposition 2

We prove existence of equilibrium by showing the existence of a fixed point of the correspondence $\mathbb{A}$. The argument is standard and makes use of Kakutani's fixed point theorem.

Proof. $\mathbb{A}$ is a self-mapping on the space of payoffs which is a subspace $X \subseteq \mathbb{R}^{n\cdot|S|}$. X is non-empty, closed, bounded and convex. Boundedness can be seen by recognizing that the maximum payoff of any player in any state is the maximum valuation across all states.

Now, $\mathbb{A}$ is single valued for most of its domain. It can be set valued for excluded players where payoffs for active players are equal for agreement and delay. In those instances the correspondence $\mathbb{A}$ maps into a closed interval which implies that the correspondence is convex. Finally, the end-points of the interval are such that $\mathbb{A}$ has a closed graph.

Then by Kakutani's Fixed Point Theorem $\mathbb{A}$ has a fixed point. $\square$

8.3 Proof of Proposition 3

We proof by contradiction. Suppose there exists a state $\tilde{s}$ such that (i) in equilibrium no agreement is truck and (ii) a decision to delay is strictly not efficient, implying that $v(\tilde{s}) > \delta w^*$.

Now, for delay to be an equilibrium outcome Proposition 1 requires that:

$$v(\tilde{s}) \leq \delta \sum_{i \in R(\tilde{s})} E_\sigma \left[f_i(s') \right] \tag{14}$$

w^* being the maximum total surplus that can be achieved under *any* stopping rule, it also is the maximum expected payoff that all players can jointly achieve, and thus it must be at least as large as the payoff available to players that are on the trade route $R(\tilde{s})$ in state $\tilde{s}$. It follows that:

$$\sum_{i \in R(\tilde{s})} E_\sigma \left[f_i(s') \right] \leq \sum_{i \in N} E_\sigma \left[f_i(s') \right] \leq w^* \tag{15}$$

Combining these terms, we get:

$$v(\tilde{s}) \leq \delta \sum_{j \in R(\tilde{s})} E_\sigma \left[f_j(s') \right] \tag{16}$$

$$\leq \delta w^* \tag{17}$$

where the final step establishes the contradiction with leg (ii) of the premise. $\square$

8.4 Proof of Proposition 5

Before we show that limit payoffs for non-essential players converge to zero, we will prove a supporting lemma.

Lemma 6. *In an SSPE of the game with equal surplus in all states, the limit payoff of any essential player i is the same in every state as $\delta \to 1$.*

Proof of Lemma 6: First, note that by Corollary 4, there is agreement in every state given that $v(s) = v \ \forall \ s \in S$. Now, let player i be essential and consider $f_i(s)$, the payoff for player i in state s. As player i is essential, the player will earn either the responder payoff or the proposer payoff with agreement in any state. As responder, the payoff to player i will be the discounted expected payoff $\delta E_\sigma \left[f_i(s')\right]$. As a proposer, the payoff to i will be $v - \delta E_\sigma \left[\sum_{j \in R(s) \setminus i} f_j(s')\right]$, which is at least as large as $\delta E_\sigma \left[f_i(s')\right]$ by the fact that there is agreement. The payoff to an essential player is thus never less than $\delta E_\sigma \left[f_i(s')\right]$ in any state, which in the limit, as $\delta \to 1$, converges to $E_\sigma \left[f_i(s')\right]$, the expected payoff to player i. As the expectations operator presents a weighted average over all states, this implies that in the limit as $\delta \to 1$ player i's payoff will converge to $E_\sigma \left[f_i(s')\right]$ in every state. $\square$

Next, we will show that Lemma 6 implies that if there exists at least one essential player the payoff for any non-essential player in any state in which they are proposing converges to the expected payoff. Consider a non-essential player j and state $\hat{s}$ such that player j is proposing in state $\hat{s}$. Let player i be an essential player that is responding in that state. Furthermore, let $\tilde{s}$ be a second state on the same route such that $R(\hat{s}) = R(\tilde{s}) = R$, but with roles reversed so that player i is the proposer in state $\tilde{s}$ and player j is a respondent.

Then, from Lemma 6 we know that in an SSPE the limit payoff to i as $\delta \to 1$ is the same across the two states $\hat{s}$ and $\tilde{s}$. Furthermore, Proposition 1 implies that payoffs for all players other than i and j, who are either responding or excluded in both states, are the same in states $\hat{s}$ and $\tilde{s}$ as well. Now, given that the available surplus is $v(\hat{s}) = v(\hat{s}) = v$ it follows that in the limit the payoff for non-essential player j is the same across the two states $\hat{s}$ and $\tilde{s}$ as well. In other words, in the limit the payoff to the non-essential player j in any state where j is proposing converges to the expected payoff $E_\sigma \left[f_j(s') \right]$.

Finally, as the expectations operator presents a weighted average over all states and by Proposition 1 player j never receives a payoff greater than in states where j is proposing, we have that the limit payoff for player j is the same across states. As j is non-essential, there exists at least one state arriving with strictly positive probability in which player j is excluded and receives a zero payoff and thus we deduce $\lim_{\delta \to 1} E_\sigma \left[f_j(s') \right] = 0$ as required. $\square$

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